thermodynamics may be used in addition to perfect gas In this regard, the characteristics approach offers a distinct advantage over any approximate method which assumes a perfect gas

It is felt that both approaches have merit, and their use is dependent upon the accuracy of the results desired and the mathematical tools available

Comment on Calculation of Oblique Shock Waves

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Nomenclature

M = Mach number upstream from an oblique shock wave
θ = acute angle between an oblique shock wave and the free-stream flow direction
δ = acute angle between a deflecting surface and the free-stream flow direction
γ = ratio of specific heats

MANY authors have pointed out the lack of simple relationships connecting the deflection angle δ and the flow across a resulting oblique shock wave [1–3]

Since most problems are specified by a deflection angle, it is necessary then to find the shock wave angle θ corresponding to the flow conditions and the specified deflection The calculation of the fluid-property changes across the shock wave, knowing the wave angle, is a simple and straightforward problem [2 4] Recognizing the need for a procedure to obtain the wave angle in terms of the parameters specifying the problem, Thompson[1] presented a cubic in $\sin^2\theta$, which reduces to the following:

$$\sin^6\theta + a\sin^2\theta + b = 0$$

where

$$a = \frac{2M^2+1}{M^4} + \left[\frac{(\gamma+1)^2}{4} + \frac{\gamma-1}{M^2}\right]\sin^2\delta - \frac{k^2}{3} \quad (1)$$

$$b = -\frac{2}{27}k^3 + \frac{k}{3}\left\{\frac{2M^2+1}{M^4} + \left[\frac{(\gamma+1)^2}{4} + \frac{\gamma-1}{M^2}\right]\sin^2\delta\right\} - \frac{\cos^2\delta}{M^4} \quad (2)$$

$$k = \frac{M^2+2}{M^2} + \gamma\sin^2\delta \quad (3)$$

Thompson further suggested that numerical procedures be used to derive the roots of this equation to any desired degree of accuracy Numerical procedures are not required, however, and the roots can be obtained in closed form The Cardan equations[4] give three roots for $\sin^2\theta$ from the foregoing equation The smallest of these roots corresponds to an entropy decrease and must be discarded immediately as being a violation of the basic laws of thermodynamics The largest root corresponds to the strong shock wave solution, and the remaining root, the case of primary interest, is the weak shock solution The weak shock root is given by

$$\sin^2\theta = -\frac{A+B}{2} - \frac{A-B}{2}(-3)^{1/2} + \frac{k}{3} \quad (4)$$

where

$$A = \left[-\frac{b}{2} + \left(\frac{b^2}{4} + \frac{a^3}{27}\right)^{1/2}\right]^{1/3} \quad (5)$$

$$B = \left[-\frac{b}{2} - \left(\frac{b^2}{4} + \frac{a^3}{27}\right)^{1/2}\right]^{1/3} \quad (6)$$

Employing the Argand representation of complex quantities and Euler's relationship, this root further reduces to

$$\sin^2\theta = -2\left(-\frac{a}{3}\right)^{1/2}\left\{\cos\left[\frac{\tan^{-1}(-2h/b)}{3}\right] + (3)^{1/2}\sin\left[\frac{\tan^{-1}(-2h/b)}{3}\right]\right\} + \frac{k}{3} \quad (7)$$

where

$$h = [(-b^2/4) - (a^3/27)]^{1/2} \quad (8)$$

Manipulation of the trigonometric functions gives as the final result

$$\sin^2\theta = \frac{k}{3} - 2\left(-\frac{a}{3}\right)^{1/2}\cos\left[60° + \frac{\tan^{-1}(-2h/b)}{3}\right] \quad (9)$$

Similarly, the root corresponding to the strong wave reduces to

$$\sin^2\theta = \frac{k}{3} + 2\left(-\frac{a}{3}\right)^{1/2}\cos\left[\frac{\tan^{-1}(-2h/b)}{3}\right] \quad (10)$$

Thus, the properties across an oblique shock wave arising from a given deflection and flow condition can be determined in closed form It requires merely the substitution of the given conditions into Eqs (9) and (10) to determine the shock wave angle This angle is then substituted into the easily derivable oblique shock relationships, which are widely available in the literature [2 3]

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Comment on "Application of Dynamic Programming to Optimizing the Orbital Control Process of a 24-Hour Communications Satellite"

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REFERENCE 1 treats the minimization of the sum of the squares of the terminal errors in three orbital parameters subject to a constraint on the amount of fuel consumed

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